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Contrast sensitivity to patch stimuli: Effects of spatial bandwidth and temporal presentation

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Abstract—Models of the spatial response of human vision are important for applied work, but the available contrast sensitivity function (CSF) data vary widely due to the diverse spatiotemporal stimuli used over the years. To assist selection, this paper: (1) reports measurements of the effects on the CSF of varying the spatial and temporal windows of grating patches; (2) demonstrates that the widely discrepant CSFs from previous studies can be accounted for by using these results; and (3) discusses simple criteria for choosing CSFs for practical applications. CSFs were measured for several combinations of spatial and temporal waveforms, using the same subjects under otherwise identical conditions. The CSF was measured over the range of 0.5–10 c/deg using Gabor-type patches of 1.0-, 0.5-, 0.25-, and 0.125-octave spatial bandwidths using both abrupt and gradual temporal presentations. The results were compared with the CSF obtained with a fixed aperture (4 deg × 4 deg) grating pattern. Increasing the number of cycles resulted in increased sensitivity at intermediate frequencies, changing the CSF to a narrower bandpass shape. For each patch bandwidth, the gradual presentation CSF had a narrower spatial pass band than with the abrupt presentation. The relevance of the large differences in the CSFs obtained with different stimuli to our understanding of visual performance is discussed.

INTRODUCTION

The spatial response of the human visual system plays an important role in models for a variety of problems in applied vision. For example, researchers have used contrast sensitivity functions (CSFs) in designing image processing algorithms, most notably for image coding (Nill, 1985; Ngan *et al.*, 1991). In most cases, the algorithm developers have not actually measured the CSFs of subjects under the relevant viewing condition, but have instead used CSFs from the psychophysical literature or secondary sources. However, the shape of the human CSF is known to depend upon a number of viewing and stimuli parameters (Graham, 1989). Over the years, a wide variety of stimulus patterns have been used to measure spatial CSFs, and the resulting curves have varied substantially. This diversity has both basic and applied consequences. Applied researchers seeking to model human spatial vision must choose a CSF. Spatial frequency bandwidth and temporal waveform are probably the two most important stimulus characteristics. In this paper we: (1) describe new measurements of the effect of varying the spatial and temporal windows of grating patches, using the same subjects under otherwise identical conditions; (2) demonstrate that the widely discrepant CSF from previous studies can be accounted for using our results; and (3) discuss simple criteria for choosing CSFs for practical applications.

When early CSFs were measured, the predominant model of visual spatial analysis

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was global Fourier analysis, which requires that the stimuli be well localized in the frequency domain. Accordingly, extended grating stimuli with many cycles generally were displayed on a rectangular screen of fixed size. Due to the fixed aperture, the number of grating cycles presented varied with the spatial frequency. The effect of this is probably negligible at high spatial frequencies, but at low spatial frequencies this produces an important confounding of spatial frequency and spatial frequency bandwidth (McCann *et al.*, 1978; Savoy, 1978). The use of stimuli containing the same number of cycles at every spatial frequency had been advocated by many investigators (Estévez and Cavonius, 1976; Howell and Hess, 1978).

More recent models of spatial vision have assumed that early spatial analyzers are localized in both the space domain and the spatial frequency domain. At the same time, lower cost, higher quality digital image processing equipment has become available, allowing widespread use of two-dimensional patches of gratings for CSF measurements. The most common has been a Gabor type function consisting of a sinusoidal grating in a two-dimensional Gaussian envelope (Swanson *et al.*, 1984; Cannon and Fullenkamp, 1988; Pointer and Hess, 1989), but other localized stimuli such as derivatives of Gaussian (Swanson *et al.*, 1984) and cosine gratings damped by half-cosines (Banks *et al.*, 1987) have been used as well. Unlike the early fixed-aperture patterns, these patch patterns have a fixed spatial frequency bandwidth. The patch patterns feature a gradual decline of stimulus contrast in both spatial dimensions, and spatial frequency is changed by scaling the entire pattern so that the number of cycles presented does not change with spatial frequency.

In addition to spatial scale (frequency), these test patterns have spatial and temporal windows, and there is no consensus yet on the most appropriate stimulus parameters (Logvinenko, 1990). Each experimenter typically has used a personal choice of bandwidth (number of cycles), ranging from as narrow as 0.18 octave (Banks *et al.*, 1987) through 0.52 octave (Watson, 1982; Davis, 1990) and 1.0 octave (Watson, 1987; Peli *et al.*, 1990), up to 2.17 octaves (Cannon and Fullenkamp, 1988).

The temporal waveform of the patterns also has varied from experiment to experiment. Continuous presentation has been used, as have gradual fade-in and fade-out (Watson, 1987; Pointer and Hess, 1989) and abrupt onset and offset (Cannon and Fullenkamp, 1988). With fixed-aperture extended gratings, flashed abrupt presentation increased sensitivity at low spatial frequencies (0.5–5 c/deg) relative to continuous presentation (Breitmeyer and Julesz, 1975) or gradual presentation (Thomas, 1987). For patch stimuli, the spatial and temporal parameters giving maximum sensitivity were determined by Watson *et al.* (1983), and Wilson (1978) presented data for transient (flashed) and sustained (gradual) temporal presentation of difference-of-Gaussian patterns. Watson *et al.* reported data for only a few combinations of spatial and temporal parameters, and Wilson's transient patterns were of a different spatial frequency than his gradual patterns. Thus, the influence of the various patch parameters on the shape of the measured CSFs as yet has not been documented adequately, preventing comparisons among studies using different stimuli.

Selection of CSF for applied vision models

Different parameters may be appropriate for various applied purposes. For some applications, the high sensitivities (300 at the peak) resulting from older methods of estimation of CSFs (fixed-size extended gratings and gradual presentation) seem suitable. Measurements of maximal threshold sensitivity may be useful in clinical situations in which the increased sensitivity may provide earlier detection of visual loss. Extended grating stimuli are also fitting for experiments in which global Fourier transform is used to analyze the results.

However, extended grating CSFs are unlikely to represent actual contrast sensitivity in free viewing of real-world images, and, therefore, may be inappropriate for vision simulation in normals (Ginsburg, 1978) and low-vision patients (Lundh *et al.*, 1981). Even though there are exceptions (such as picket fences), periodic high-contrast patterns rarely occur in natural scenes. Thus, the sensitivity added by spatial summation over many cycles is unlikely to be representative of visual perception of images other than gratings. One needs a reasonable criterion as to the most valid or natural amount of spatial summation to allow in CSF measurements. A number of investigators (e.g., Watson, 1982) have argued that 1 octave is an appropriate estimate of the spatial frequency bandwidth of the underlying mechanisms.

Similar arguments can be made about the choice of temporal presentation. In many applied settings the observer's saccadic scan-path pattern of eye movements results in abrupt changes of eye position. Therefore, thresholds measured with abrupt presentation of grating patches may be more suitable for applied work than those measured with gradual fade-in and fade-out.

CSFs typically have been measured using paradigms in which the subject reports only the presence of the target (detection), without the need for perception of spatial form, such as orientation, profile, or contrast. Even though detection is generally more sensitive than discrimination, it is not necessarily the measurement most representative of useful image perception. For this purpose, one's threshold task should include some aspects of form discrimination (Fleck, 1989). For this reason, we chose to measure CSFs for the discrimination of orientation of horizontal and vertical gratings (Peli *et al.*, 1990).

METHODS

Stimuli

Patches of sinusoidal gratings in a Gaussian envelope (Gabor functions) were flashed for 1 s. We used bandwidths of 0.25, 0.5, and 1.0 octave and abrupt and gradual stimulus onset and offset. Stimuli were generated with an Adage 3000 image processor with 10-bit digital-to-analog converters and displayed on a Tektronix 690SR, 60-Hz noninterlaced monitor. The calibrated display provided a linear response over 3 log units of luminance (Arend and Reeves, 1986). Watson *et al.* (1986) described calibration of a similar system. The mean luminance of the display was 46 cd/m^2.

The Gabor function stimuli were composed of horizontal or vertical sinusoidal gratings in cosine phase, multiplied by a two-dimensional Gaussian envelope. The spatial luminance distribution of the vertical patch is described as:

$$p(x, y) = L_0\left[1 + m\cos(2\pi f_0 x)\exp\left(-\frac{x^2+y^2}{\sigma^2}\right)\right],$$

where L_0 is the mean luminance, m the nominal contrast, and σ the spatial spread of the Gaussian envelope. (This slightly unconventional expression for the spatial spread of the Gaussian simplifies our expressions for the relations among patterns presented in the Appendix.) The horizontal patch had the same waveform, but rotated 90 deg.

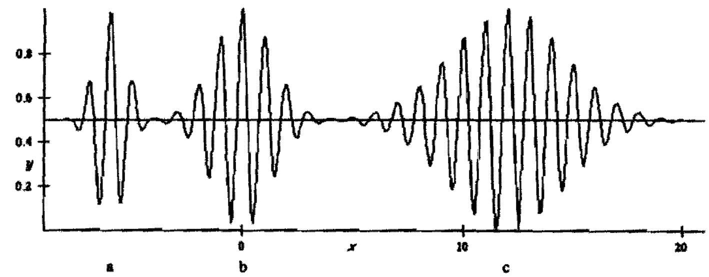

Figure 1. Luminance profile through the center of Gabor patch stimulus patterns. (a) 1.0-octave bandwidth; (b) 0.5-octave bandwidth; (c) 0. 25-octave bandwidth.

The spatial frequency amplitude spectrum of our stimulus is also a Gaussian, centered at frequency f_0. We define the spatial frequency bandwidth as the ratio, in octaves, of the spatial frequencies at which the spectrum drops to $1/e$ (i.e., 37%) of its value at f_0. Another common bandwidth definition is full width at half amplitude. The relations among the various parameters in the spatial frequency domain and in the space domain for both conventional bandwidth definitions are presented in the Appendix.

Due to the rapid decline of the Gaussian envelope (Fig. 1), the pattern was distinguishable from the mean luminance surround only within about a 2σ radius of the envelope. Consequently, we calculated and presented our patch stimuli to a radius of 2σ. For the 1-octave stimuli, where $\sigma = 3/\pi = 0.955$ cycle, only about two cycles were visible even at high suprathreshold contrasts. Our 0.5- and 0.25-octave patches spanned about four and eight visible cycles, respectively. The cosine phase Gabor patches have a small d.c. component that varies with the bandwidth (Cannon and Fullenkamp, 1988). However, the value of this d.c. component for our stimuli (1-octave and narrower) is negligible.

CSFs also were measured with extended sinusoidal gratings presented within a 4 deg $\times$ 4 deg square aperture. The surrounding screen was set to the grating mean luminance, 46 cd/m^2. We refer to CSFs measured with these stimuli as the fixed-aperture CSFs.

Procedure

Subjects viewed the monochrome CRT display from a distance of 80 in. The subjects, using a chin rest, viewed the screen with both eyes, through their natural pupils. A low-contrast fixation point was provided at the center of the screen to aid fixation and accommodation. Each session began with 5 min of dark adaptation followed by 5 min of adaptation to the mean luminance of the screen. In all experiments, stimuli were presented in a two-alternative, forced-choice paradigm. Seven spatial frequencies from 0.5 to 10 c/deg were randomly interleaved within each session. When ready, the subject pressed a button to present a grating patch. In the abrupt-presentation condition the stimulus was displayed for 1.0 s, with abrupt onset and offset. In the gradual-presentation condition the stimulus contrast increased linearly for 0.25 s, remained at maximum contrast for 0.5 s and decreased linearly for 0.25 s. The subject then pressed buttons to indicate whether the patch was horizontal or vertical and received auditory feedback.

The psychophysical procedure was a hybrid method consisting of three successive phases beginning with a staircase procedure, with contrast changing in 0.1-log unit steps. After the second reversal of direction, the program continued to set the contrast by the staircase, but it began analyzing the accumulating data with a modified parameter estimation by the sequential testing (PEST) method (Lieberman and Pentland, 1982). When the running threshold estimate from the PEST analysis met a minimum reliability criterion (i.e. within a confidence interval $>40\%$), stimulus control was given to the PEST algorithm for the last block of trials, until the accuracy criterion for termination was met. This hybrid method prevented the long random walks that occur occasionally at the beginning of a PEST routine (Klein and Manny, 1989). With this method, 50 to 80 presentations were required to reach the termination criterion. A Weibull psychometric function then was fitted to the data to obtain the threshold estimate. The PROBIT analysis also provided a sampling statistic for the threshold, $\widehat{se}$, that is comparable to the standard error of the mean. It is derived from the curvature of the probability surface at its maximum (the threshold point), on the assumption that the base parameters (threshold and steepness of the psychometric function) are normally distributed.

Subjects

Two groups of subjects, young adults with normal or corrected-to-normal visual acuities, participated in the study. Informed consent was obtained from each before testing. Group 1 ($n = 3$) was tested with all the stimuli. Group 2 ($n = 14$) was tested with only the 1-octave patches and the fixed-aperture grating stimuli.

RESULTS

The Group 2 ($n = 14$) individual CSFs for both the 1-octave patches and the fixed-aperture gratings are presented in Fig. 2. The abrupt presentation was used for both patterns. For all subjects, clear differences existed between the patch and fixed-aperture grating CSFs. The former were lowpass, over the range of frequencies tested; the latter had a bandpass shape, peaking at about 4 c/deg. The sensitivity to patch stimuli was 0.5–1.0 log unit lower than fixed-aperture grating sensitivity over most of the spatial frequencies measured.

Figure 3 shows the averaged results from Group 1 ($n = 3$) using three different patch bandwidths (1, 0.5, and 0.25 octave), both with the abrupt (filled symbols) and gradual (open symbols) temporal presentations. The individual results for two of the subjects are illustrated in Fig. 4. The third observer gave similar data, but the changes were smaller in magnitude. The lowest spatial frequencies could not be presented for the narrower bandwidth stimuli due to the limited width of the screen; hence, the lowest frequencies tested were different for each bandwidth.

The change with the increasing number of cycles represents increased sensitivity at intermediate frequencies. With abrupt presentation (Fig. 3a), the shape of the CSFs changed gradually from lowpass to bandpass as the number of cycles increased. There was also an overall increase in sensitivity with increased number of cycles. A similar increase in sensitivity with decrease in bandwidth was reported by Cannon and Fullenkamp (1988) for their 4 c/deg Gabor patches. With the gradual temporal presentation (Fig. 3b), the CSFs showed a similar tendency toward bandpass characteristics (decreased response bandwidth) with decreased stimuli spatial frequency

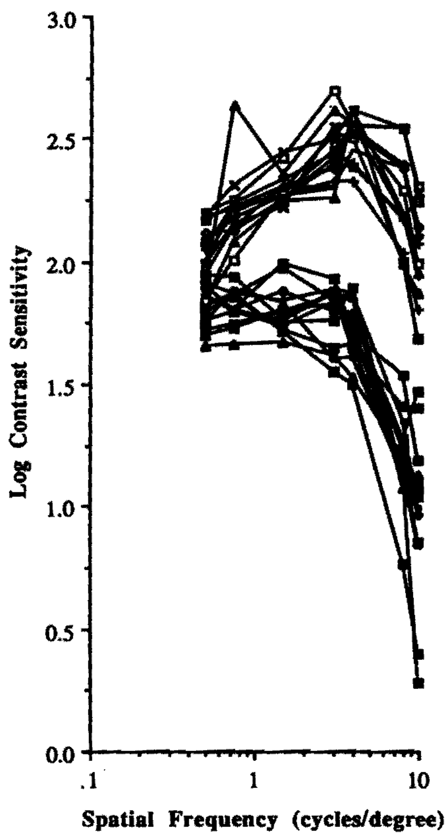

Figure 2. Individual contrast sensitivity functions of 14 normal, young observers for two types of stimulus patterns. Upper curves: 4 deg × 4 deg fixed aperture. Lower curves: 1-octave-bandwidth Gabor patches. Despite large individual differences the data for the two different stimuli clearly form two distinct groups.

bandwidth. At each patch bandwidth, the pass bands of the gradual-presentation CSFs were slightly narrower than those for abrupt presentation (Fig. 3c). The differences between the CSFs for abrupt and gradual temporal presentation were smaller for the narrower bandwidth stimuli. The large increase in sensitivity for gradual presentation at intermediate frequencies for the 1-octave patch is especially noteworthy, because the integrated energy of the gradual stimuli was lower than that of the abrupt stimuli.

For all bandwidths tested, the gradual temporal presentation resulted in narrower bandpass characteristics stemming from both increased sensitivity at intermediate frequencies and decreased sensitivity at low frequencies. For the 1- and 0.5-octave patch stimuli, gradual presentation resulted in a significant increase in sensitivity at the intermediate frequencies of 3 and 4 c/deg, relative to the abrupt curve. For all three subjects, the sensitivity to the gradual presentation was actually lower than the abrupt curve at our lowest spatial frequency, 0.5 c/deg.

Results with 0.125 octave were measured for one subject (Fig. 4a), and were similar

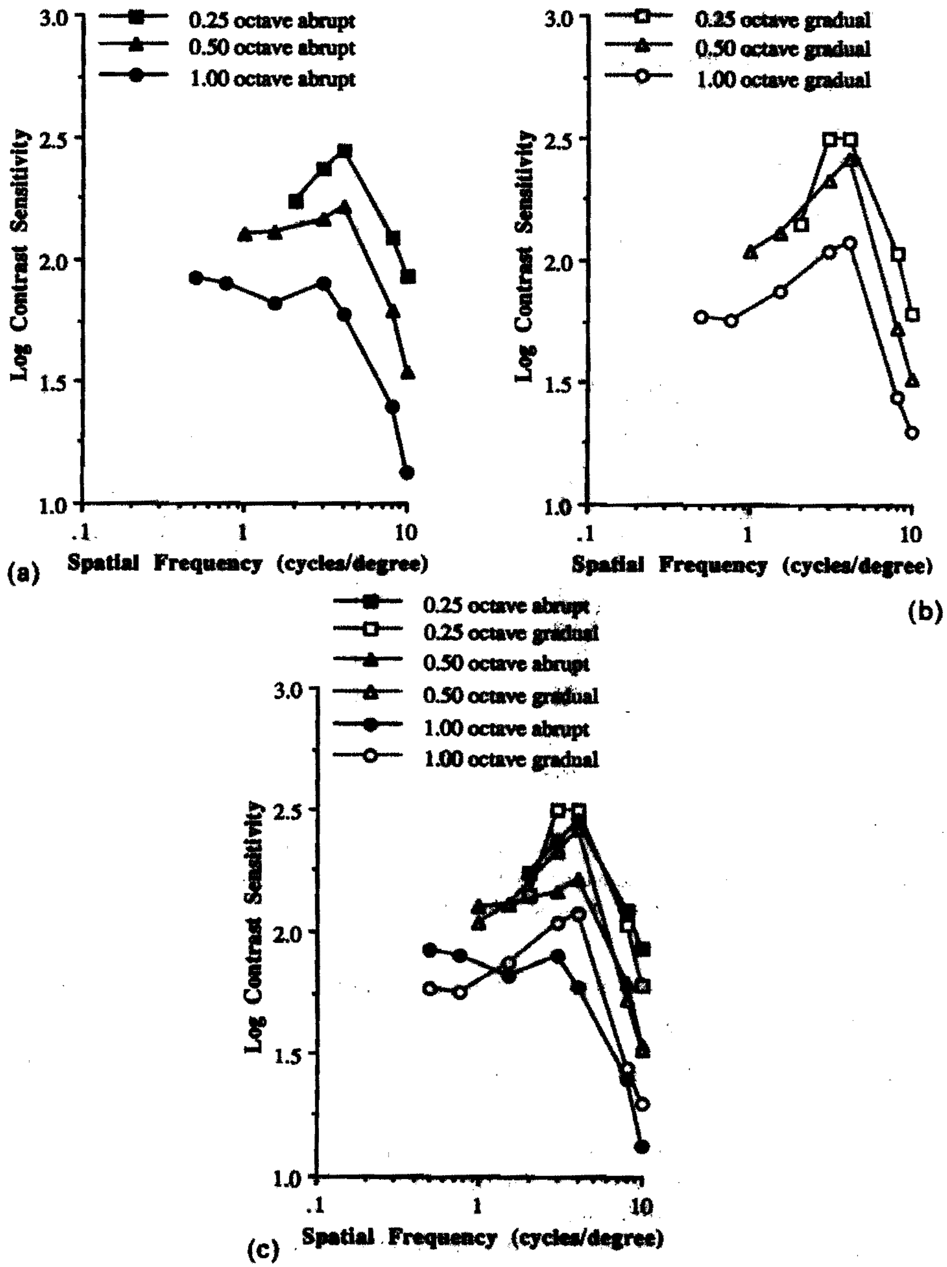

Figure 3. The effects of spatial bandwidth and temporal waveform on the Gabor-patch contrast sensitivity functions. Data are the averaged results from Group 1 (n = 3). (a) Solid lines, filled symbols: abrupt onset and offset (1-s duration). (b) Dashed lines, open symbols: gradual (linear ramp) onset and offset. (c) The results from (a) and (b) are combined to facilitate appreciation of the effect of temporal presentation.

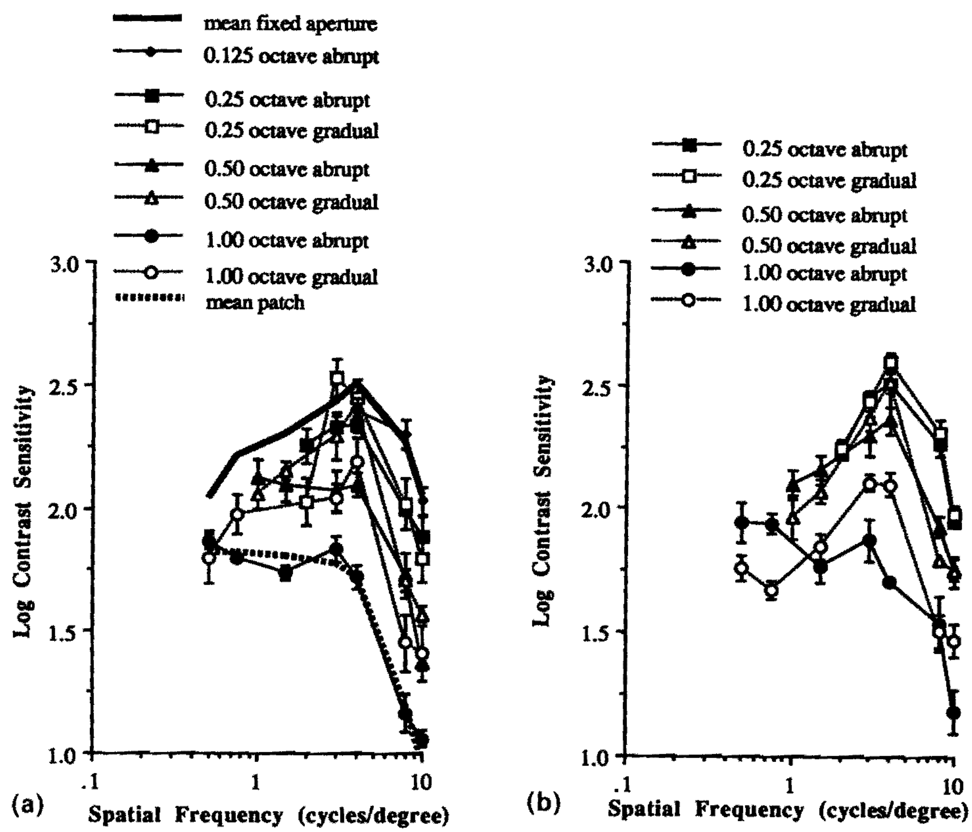

Figure 4. The effect of bandwidth and temporal waveform on the Gabor-patch contrast sensitivity functions (CFSs) of two subjects. (a) One subject compared with the mean CSFs of data illustrated in Fig. 2. CSF for 0.125 octave is from the same subject. (b) Similar data for a second observer. Error bars: ± 1 se. (See text for explanation of derivation of the sampling error).

to the results with 0.25 octave, showing the same tendencies of narrowed bandpass and higher peak sensitivity.

COMPARISONS AND ANALYSIS OF PREVIOUS DATA

Agreement

Our results may enable comparisons among CSFs from different studies using different temporal and spatial windows for the grating stimuli. To determine whether such comparisons are possible, we replotted CSFs from a number of studies and compared their relationships in light of our results. Cannon (1985) (foveal data assembled from all his figures) and Thomas (1987) (his Table 1 subject KS) used fixed-aperture stimuli of 2 deg and 3 deg diameters, respectively. The temporal presentations were gradual (2 s) and abrupt (1 s), respectively. Their results, replotted in Fig. 5, are in excellent agreement with our fixed-aperture CSF, regarding the general shape and the values of the sensitivity. The effects of temporal presentation on low-frequency sensitivity are too small to compare across studies with other differing

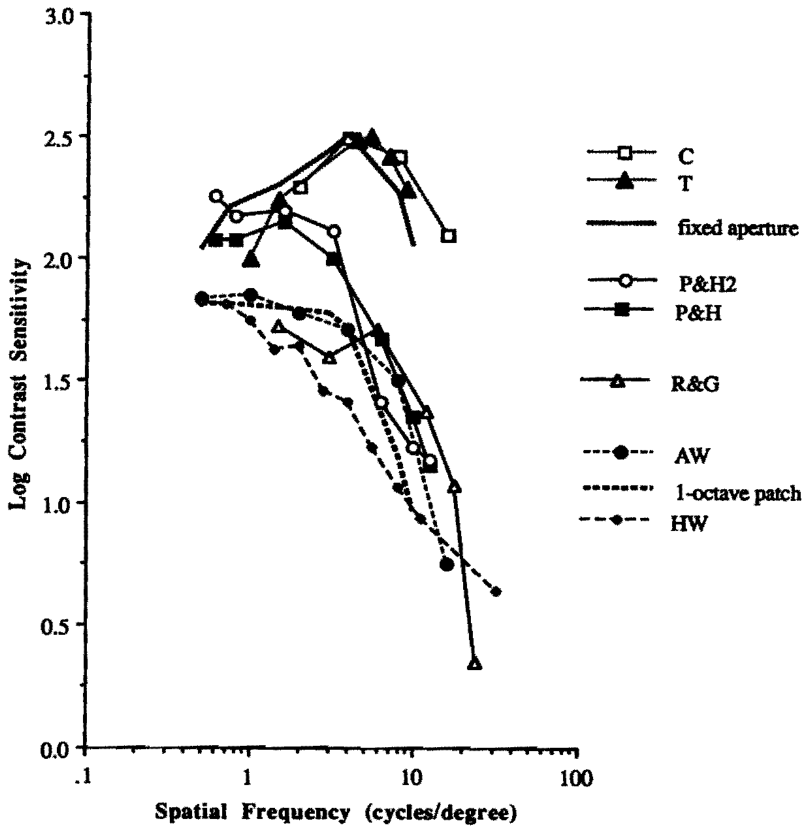

Figure 5. Comparison of contrast sensitivity functions (CSFs) from various studies demonstrates a large measure of agreement among CSFs obtained with various psychometric methods when the spatial and temporal bandwidths are accounted for. Each line in our graph represents data from one study. 'C' data from Cannon (1985) using a 2 deg fixed aperture with gradual presentation of 2 s, 'T' from Thomas (1987) using a 3 deg fixed aperture presented abruptly for 1 s. 'P & H2' and 'P & H' are from Pointer and Hess (1989) using a 0.24-octave patch presented in a Gaussian temporal envelope of 250-ms spread. 'R & G' is from Robson and Graham (1981) using a 0.4-octave patch of grating presented briefly (temporal spread 100 ms). Data for 'AW' are from Watson (1987) who used Gabor patches of 1.2-octave bandwidth ($1/e$) presented in a Gaussian temporal window of 250-ms temporal spread. 'HW' is from Wilson (1978), a wide (>2 octave) patch presented abruptly and briefly. The '1-octave patch' and 'fixed-aperture' results are reproduced from our Fig. 4.

parameters and psychometric methods. However, Thomas (1987) did report measuring this effect across two of his subjects. Watson (1987) (his Fig. 3) used Gabor patches of 1.2-octave bandwidth ($1/e$) presented in a Gaussian temporal window of 250-ms temporal spread. His results are almost identical to our 1-octave abrupt-patch CSF (Fig. 5). In both cases, the function is lowpass in character, and the absolute sensitivity is similar over most of the frequency range. The similarity of these results may be due to the higher spatial bandwidth and narrower temporal bandwidth of his stimuli, which could have canceled each other. Wilson (1978) (his Fig. 8) used transient patch stimuli composed of one-half a cycle of cosine presented abruptly and briefly. Such a stimulus has a spatial bandwidth wider than 2 octaves, and thus would be expected to result in even lower sensitivity and more of a lowpass character than the 1-octave patch (Fig. 5).

The previous work used detection tasks. Our task was orientation discrimination. In many cases, detection and discrimination thresholds vary in similar ways as a function of various variables (Derrington and Henning, 1981; Thomas, 1985). Therefore, conclusions derived from our data probably apply also to detection CSFs (see Fig. 5 for close agreement between data obtained in detection paradigms and our own).

Discrepancies

Pointer and Hess (1989) used Gabor patches of 3.2 cycles spatial spread, corresponding to a 0.24-octave bandwidth ('P & H2' and 'P & H' are from their Figs 2 and 3, respectively). Their results are intermediate to the results discussed above. The sensitivities they measured with a Gaussian temporal envelope of 250-ms spread are lower than those we found for similar stimuli (using a presentation 4 times longer) for the intermediate and high frequencies, but they do exhibit a more bandpass character in agreement with our results. Robson and Graham (1981) (their Fig. 4) used a 0.4-octave patch of grating presented briefly (temporal spread 100 ms). Their results (Fig. 5) have the same shape as our 0.5-octave abrupt results (Fig. 3), but are about 0.5 log unit lower. This reduction in sensitivity may be due to the brevity of the temporal presentation. It should be emphasized that the differences between the results of the various studies and our results are not larger than the range of variability we found among our own subjects' responses (Fig. 2).

Our results appear to differ from those of two previous studies. Estévez and Cavonius (1976) found no change in the shape of the CSF (which peaked at 0.8 c/deg) with an increased number of cycles over the same range as ours. Further, the increased sensitivity as the bandwidth decreased was much smaller in their CSFs than in ours. However, these discrepancies may be more apparent than real. The previous study used fixed-height, rectangular-aperture gratings, presented continuously. Continuous presentation probably corresponds better to our gradual temporal presentations than to our abrupt. In the gradual condition, we also found that the CSFs for various bandwidths had similar shapes, and the increased sensitivity with an increasing number of cycles was smaller than for abrupt presentation. Estévez and Cavonius (1976) did find that bandwidth affected sensitivity more when the grating was surrounded by a dark mask. However, our stimuli did not have a dark surround. Howell and Hess (1978) concluded from their CSFs that the functional summation of responses of detecting elements at threshold extends over a constant number of pattern cycles, implying that the effect of pattern bandwidth was independent of frequency. By their logic, our results imply a relative increase in summation area (in cycles) for intermediate spatial frequencies. It should be noted that our Group 2 data are consistent with our Group 1 results in this regard.

Previous studies did report an increase in the bandpass characteristics of the CSF with gradual temporal presentation as compared with abrupt presentations (Breitmeyer and Julesz, 1975; Thomas, 1987). However, in these and other similar studies, this change was associated only with decreased sensitivity at low spatial frequencies for gradual temporal presentations, even though we also found a large increase in sensitivity at intermediate frequencies for wide band patches. For narrow band stimuli, such as those used in previous comparisons of temporal presentation, a ceiling effect obscures this increase in sensitivity at intermediate frequencies.

CONCLUSION

The many combinations of parameters that have been used with localized patch stimuli have prevented comparison of results across studies. We believe that the relationships described here can be helpful in this regard.

The large differences among the CSFs obtained with various patterns raise two questions: (1) Is there one type of stimulus that is more appropriate for a specific application? and (2) What is the contribution of the different CSFs to our understanding of visual perception in various applied settings?

It would be useful to have an intuitive grasp of the implications of these CSF differences for perception of local contrast in complex images. Most efforts to apply laboratory measures of spatial response to practical problems have been based on concepts of linear filters and convolution. Unfortunately, those global filter concepts are of little value in complicated images. Under those conditions, it is not sufficient merely to treat the CSF as a linear filter. The linear systems approach ignores several nonlinearities of normal vision that are important for image appearance: (1) Information that is below threshold is not merely highly attenuated, it is lost and cannot be recovered. (2) Curves of constant suprathreshold apparent contrast are not multiples of the CSF. Apparent contrast is relatively independent of retinal eccentricity and spatial frequency (contrast constancy), even though the contrast threshold changes significantly (Cannon, 1985). (3) Thresholding should be applied to quasi-local image contrast, which is a nonlinear function of the local amplitudes in the image.

Therefore, we need some alternative to the linear analysis that takes these nonlinearities into account. Simulations can provide such an alternative. In this context, a simulation is a set of images computed in such a way that comparisons of their appearances to the viewer illustrate some distinction. The distinction may be among two or more CSFs or differing viewing conditions (e.g., distance, retinal eccentricity, optic media).

We have computed such simulations, using a nonlinear procedure (Peli, 1990; Peli *et al.*, 1991) that avoids criticisms of previous linear simulations (Tyler, 1977). The simulations based on our patch CSF and our fixed-aperture CSF have been published elsewhere (Peli *et al.*, 1990). The results presented there lent further support to the above arguments that the 1-octave patch CSF provides a better description of the visibility of image detail for normal observers than the fixed-aperture CSF. At the simulated observation distance and farther away, the images obtained using the patch CSFs were indistinguishable from the original image. At any shorter distance, the difference became apparent. The simulation generated with the fixed-aperture CSFs may be distinguished from the original image only at about half the simulated observation distance. These simulations also demonstrated that under certain conditions, large differences in CSF used (up to 1 log unit) may result in a relatively small effect on a simulated image appearance.

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APPENDIX

Comparison of grating-patch bandwidths

The conventional expression for one-dimensional Gaussian-weighted cosine patterns is

$$\cos(2\pi f_0 x)\exp\left(-\frac{x^2}{2\sigma_x'^2}\right). \tag{A1}$$

The bandwidth expressions are simpler if we make the following substitution for the width of the Gaussian

$$\sigma_x^2 = 2\sigma_x'^2, \tag{A2}$$

so that the Gaussian and its Fourier transform take the form (Castleman, 1979):

$$\exp\left(-\frac{x^2}{\sigma_x^2}\right) \Leftrightarrow \sigma_x\sqrt{\pi}\exp\left(-\frac{\sigma_x^2\omega^2}{4}\right), \tag{A3}$$

where $\omega = 2\pi f$. Thus:

$$\exp\left(-\frac{x^2}{\sigma_x^2}\right) \Leftrightarrow \sigma_x\sqrt{\pi}\exp(-\sigma_x^2\pi^2 f^2) \equiv \sigma_x\sqrt{\pi}\exp\left(-\frac{f^2}{\sigma_f^2}\right). \tag{A4}$$

Therefore,

$$\sigma_x = \frac{1}{\pi\sigma_f} \tag{A5}$$

is the relationship between the width of the space-domain and frequency-domain Gaussians.

We want the spatial frequency bandwidth B defined in octaves. The relationship between B and the number of the sinusoidal cycles in the patch depends on which definition of bandwidth we use. There is no reason to prefer the full width at half-amplitude definition over the $1/e$ definition of bandwidth. Even though some investigators used the half-amplitude definition in the spatial frequency domain

(Watson, 1982; Swanson *et al.*, 1984; Davis, 1990), others used the $1/e$ definition (Cannon and Fullenkamp, 1988). However, all of the studies we examined used the $1/e$ definition of spatial spread to determine the number of cycles.

$1/e$ height full bandwidth. The Gaussian falls to $1/e$ at a distance of σ_f from its peak. Thus

$$\frac{f_0 + \sigma_f}{f_0 - \sigma_f} = 2^B \tag{A6}$$

where f_0 is the frequency of the underlying sinusoidal grating or the mean frequency of the Gabor patch. Therefore,

$$\sigma_f = \frac{f_0(2^B - 1)}{(2^B + 1)} = \frac{1}{P}\frac{(2^B - 1)}{(2^B + 1)} \tag{A7}$$

where P is the period of a sinusoidal grating of frequency f_0. The space domain bandwidth (in gratings periods) is, therefore,

$$\sigma_x = \frac{1}{\pi\sigma_f} = \frac{P(2^B + 1)}{\pi(2^B - 1)}. \tag{A8}$$

For a 1-octave patch, for example,

$$\sigma_x = \frac{3}{\pi}P. \tag{A9}$$

Half-maximum-height full bandwidth. For this definition, we must first calculate the distance to the half-height of the envelope, i.e., where

$$\exp\left(-\frac{f^2}{\sigma_f^2}\right) = \frac{1}{2}, \tag{A10}$$

$$-\frac{f^2}{\sigma_f^2} = \ln 1 - \ln 2, \tag{A11}$$

$$f^2 = \sigma_f^2 \ln 2, \tag{A12}$$

$$f = \sqrt{\ln 2}\sigma_f. \tag{A13}$$

Thus, for B octaves of bandwidth, we have

$$\frac{f_0 + \sqrt{\ln 2}\sigma_f}{f_0 - \sqrt{\ln 2}\sigma_f} = 2^B. \tag{A14}$$

As for Eqn (A8) we can solve for σ_x:

$$\sigma_x = \sqrt{\ln 2}\,\frac{P(2^B + 1)}{\pi(2^B - 1)}, \tag{A15}$$

or, as was shown by Watson (1982) and Kulikowski *et al.* (1982),

$$B = \log_2 \frac{\dfrac{\pi\sigma_x}{P} + \sqrt{\ln 2}}{\dfrac{\pi\sigma_x}{P} - \sqrt{\ln 2}}. \tag{A16}$$

Comparing Eqns (A15) and (A8) we find that the space-domain spread (in gratings periods) differs by a factor of $\sqrt{\ln 2} = 0.8325$ between the two definitions.